

Financial mathematics I

The idea of Gauss

An anecdote from the 3rd year of Carl Friedrich Gauss (1777-1855)'s elementary school days : His teacher at the time, J. G. Büttner, punished the wild young Carl by giving him the task of adding up all the numbers from 1 to 100 - if he had really added up all the numbers one after the other, Carl would have been busy for a while. But he came up with the right solution after only a few minutes....

$$1 + 2 + 3 + \dots + 98 + 99 + 100 = 5050$$

How did he calculate it? He wrote the sum in reverse order underneath and added the two together:

$$\begin{array}{r} 1 + 2 + 3 + \dots + 98 + 99 + 100 \\ 100 + 99 + 98 + \dots + 3 + 2 + 1 \\ \hline 101 + 101 + 101 + \dots + 101 + 101 + 101 \end{array}$$

THIS INGENIOUS IDEA ONLY WORKS IF:
EACH STEP IN THE SUM IS EQUAL !!!

A sequence of numbers with this property (each step is equal) is called *a linear sequence* or *a arithmetic sequence*.

Exercises

- (1) Use this idea to calculate the sum from 1 to 1000.
- (2) In a large concert hall, the first row of seats has 30 seats, the second 32, the third 34 and so on, always two seats more until the last row with 100 seats.
 - (a) How many rows of seats does this concert hall have?
 - (b) How many seats are there in the hall?
- (3) A philatelist discovers the stamp machine at the post office. This machine accepts coins (-.05, -.1, -.2, -.5, 1.-, 2.- and 5.-) up to a maximum amount of 9.95. You insert the money and press the green button, then the machine prints the corresponding stamp. This machine can thus print all stamps from 0.05 to 9.95 in five-centime increments. Our friend wants this complete set of stamps for his collection and gets to work immediately.
 - (a) How many different stamps can this machine print?
 - (b) What is the value of this stamp collection?
 - (c) How many of which coins do we need if we always insert the largest possible coins? (Attention with the twenty-cent pieces and with the two-franklers !)

Investments during the year

Investments made during the year bear interest pro rata, i.e. according to the time of year. For example, if money is invested one trimester ($\frac{1}{4}$ year) at an annual interest rate i , the interest rate is $\frac{1}{4}i$.

Exercises

- (4) For these assignments, each month is the same length and the interest rate $i = 2\%$ *p.a* applies to all currencies.
- (a) Goretí Rodrigues Costa invests CHF4,200 for a trimester, how much interest does she get?
 - (b) Ulrich Abgottspon invests CHF885 for 10 months, how much interest does he get?
 - (c) Suniti Sugaya puts ¥30000 into her savings account at the beginning of each month. How much money is in this account after the interest is credited at the end of the year?
 - (d) How much money would Ms. Sugaya have in her account at the end of the year if she had deposited her savings on the last day of each month?
 - (e) Mosta Guerilovska had €120'000 in her account on 1.1.2019. How much interest was credited to her account at the end of the year?
 - (f) Mrs. Guerilovska withdrew €4800 on 1.1.2020. How much will she have on her account at the end of the year after the interest credit?
 - (g) Hiro Lokito will have saved exactly CHF100,000 with the interest credit at the end of 2020. What is the maximum amount he can withdraw on 1.1.2021 if he wants to have CHF100,000 back in his account on 31.12.2021 after the interest credit?
 - (h) Ika Tsokhim is saving and pays 6'000 Rupees into her account four times at the beginning of each trimester. How much has she saved at the end of the year after the interest credit?
 - (i) Her sister Norla pays 2'000 Rupees at the beginning of each month. How much does she have at the end of the year after the interest is credited?
 - (j) And her father Nuri Tsokhim saves 2,000 rupees at the end of each month. How much does he save in one year with the interest credit?
 - (k) Lisa Salaté inherited a credit balance of CHF420,000. She draws a pension supplement of CHF800 on the 1st of each month. How much remains after one year after the interest credit?
 - (l) Mrs. Salaté would like to have CHF420'000 back in her account at the end of the year. What is the maximum amount she could withdraw on the 1st of each month?
 - (m) How much could Mrs. Salaté withdraw at the end of each month if her balance of CHF420,000 is not to shrink?

- (n) Diego Schnidrig opens an account on 31.1. and makes his first deposit. How much does he have to deposit at the end of each month that he has saved CHF10'000 at the end of the year after the interest credit?

Lösungen

- (1) 500'500
- (2) (a) 36
(b) 2'340
- (3) (a) 199
(b) 995.-
(c) $-.05*100, -.1*80, -.2*160, -.5*100, 1.-*80, 2.-*160, 5.-*100$
- (4) (a) CHF21.-
(b) CHF14.75
(c) ¥363'900
(d) ¥363'300
(e) €2'400
(f) €119'952
(g) CHF1'960.75
(h) 24'300 Rupees
(i) 24'260 Rupees
(j) 24'220 Rupees
(k) CHF418'696.-
(l) CHF692.45
(m) CHF693.60
(n) CHF825.80

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