

# Proportionality

The quotient of two quantities  $\frac{a}{b}$  is called the **ratio** between  $a$  and  $b$ . A ratio is a fractional expression whose value can be expressed in decimals.

**Example 1:** How to compare the fuel consumption of different cars?

	distance (km)	consumption (l)		
red car	18'500	1'517		
wite car	26'000	2'366		
blue car	32'500	2'340		
yellow car	12'600	1'323		

a. Calculate the fuel consumption per 100 km for each of these four cars.

b. Compare the fuel consumption of these cars with that of the most economical one.

**Note:** The smallest value is not necessarily chosen as unit. Everything depends on what is important.

A **proportion** is an accordance of two ratios:

$$\frac{a}{b} = \frac{c}{d}$$

Ratios are often used to provide a basis of comparison for different processes or services.

The **rule of three** is the solution method, with which a missing expression of a proportion can be found. The difficulty is to set up the proportion. The algebraic operations are elementary.

**Example 2:** You want to buy a package of your favorite breakfast cereals. There are two sizes available: a 375g pack and a 650g pack. How much should the large package cost if the small one costs CHF 4.30?

A **Percentage** is a ratio whose denominator refers to 100. Thus the ratio can be expressed in decimal notation and the result can be multiplied by 100. The ratio  $\frac{2}{3}$  corresponds to  $\frac{66.\bar{6}}{100}$  and is written  $66.\bar{6}\%$ .  $\frac{2}{3}$  and  $66.\bar{6}\%$  are thus equally the same!

**Example 3:** A few years ago, MGB used a questionnaire to obtain information on customer behavior. 23,245 questionnaires were distributed, 13,132 returned filled out. The evaluation showed that 5,422 women and 7,710 men participated. 2'788 persons were between 16 and 24 years old and 8'692 were over 60 years old.

a. Calculate the response rate, i.e. the percentage of customers who participated in the survey.

b. What was the percentage of women who completed the questionnaire?

c. How high was the percentage of travelers in the above mentioned age groups?

**Rates** are used everywhere: Interest rate, discount rate, growth rates and also exchange rates are examples. It is the ratio of two quantities, which can be of different or the same nature. Rates are expressed as percentages or decimal numbers, depending on the situation.

**Example 4** You have had repairs done on your car. The garage owner charges CHF 210 for two and a quarter hours of work. Calculate his hourly rate.

**Example 5** You want to have your stereo system repaired. The hourly rate of the technician is CHF 80. You are informed that the repair will take two and a half hours. How much do you have to pay for the repair?

**Example 6** One company invested CHF 44'000 at an annual interest rate of 3.5%. The interest is added to the credit balance.

a. What is the amount after one year?

b. What is the amount after two years?

c. What is the amount after  $n$  years?

A **exchange rate** is the ratio between two currencies.

**Example 7** You have planned a trip to the USA and have CHF 2'500 at your disposal. You want to exchange this money into US\$. The exchange rate (sale) in the exchange office is 0.9127 CHF/US\$.

a. How many US Dollars do you receive?

b. After your return you will have US\$ 425 and the exchange rate (purchase) is now 0.9531 CHF/US\$. How many Swiss Francs will you receive?

**Example 8** You are returning from a trip around the world and still have foreign currencies (€122.-, ¥5318.- and £288.-) that you would like to exchange into Swiss francs. On the Internet you can find for the purchase:

Country	CHF/Currency	Currency/CHF
Australia	0.66507	1.50360
Brazil	0.17208	5.81125
Euro	1.08213	0.92410
Denmark	0.14541	6.87711
Great Britain	1.19859	0.83431
Hongkong	0.11837	8.44809
India	0.01244	80.38585
Japan	0.00864	115.74074
Canada	0.69689	1.43495
Malaysia	0.22011	4.54318
Mexico	0.04223	23.67985
New Zealand	0.61051	1.63797
Norway	0.10148	9.85416
Philippines	0.01885	53.05040
Sweden	0.10421	9.59601
Singapore	0.67008	1.49236
South Africa	0.05438	18.38911
Turkey	0.12263	8.15461
Czech Republic	0.0438	22.831
USA	0.91743	1.09000

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How many Swiss francs do you receive?

The **growth rate** is the ratio of the increase in value per unit time compared to the initial value.

$$\text{Growth rate} = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}}$$

The **depreciation rate** is the ratio of the loss of value per unit of time compared to the initial value.

$$\text{Depreciation rate} = \frac{\text{Initial value} - \text{Final value}}{\text{Initial value}}$$

**Example 9** Last year you bought a computer for CHF 2'299. Today you have sold it for CHF 1'050. Calculate the depreciation rate.

## Exercises

- (1) Find the missing values in the following proportions:  
 a.)  $\frac{x}{12} = \frac{3}{5}$       b.)  $\frac{2}{x} = \frac{9}{7}$       c.)  $\frac{7}{3x} = \frac{21}{5}$       d.)  $\frac{3}{7} = \frac{2x}{5}$
- (2) You work in a boutique that pays you a commission of 0.8% of the turnover in addition to your salary. How much commission do you expect if the turnover was CHF 1'850?
- (3) You bought a new car last year for €16'500.-. Today you can sell it for €12'000.-. How much percent do you have to write off?
- (4) In the clearance sale your favorite Cravatte is 25% reduced. How much do you have to pay now, if the price was €45.99.-?
- (5) In order to promote its products, a chocolate company decides to increase the size of its packages by 33% for the same price. How many chocolates will there be in the promotion pack when there are usually 6 chocolates in the pack?
- (6) A student has achieved the following points (points/max points) in the first exams of the semester: Math 21/28, Franz 48/58, English 53/64 and in Accounting 47/112.  
 a. What notes does this give in the usual 1-6 scale of notes?  
 b. What grade must he or she achieve in the second exam if he or she wants to have at least a 5 in each subject in the report card?
- (7) The tax on a residential building is CHF 16.9 per thousand of the taxable value of the property. This was set at CHF 92'500. How much must be paid to the tax authorities each year?
- (8) In 2016 5176 children were born in Zurich. Calculate the birth rate (number of births per 1000 inhabitants) if Zurich had 415'682 inhabitants in that year.
- (9) On a business trip to the USA you needed US\$ 2'650. The first CHF 1'000 you changed at a rate of 1.0944 \$/CHF, the second CHF 1'000 at 1.1012 \$/CHF and the third thousand at a rate of 1.1107 \$/CHF. Today you change the remaining US\$ into CHF. At the bank the following exchange rates for the US\$ are available:  
 Purchase: 0.8663      Sale: 0.9142  
 a. How much CHF do you have left? b. How much did you pay on average for one US\$?
- (10) A plumber charges you CHF 73.50 for his work, which lasted  $1\frac{3}{4}$  h.  
 a. How much would you have to pay if he only needed  $\frac{3}{4}$  h?  
 b. Your friend paid CHF 94.50 at the same hourly rate. How long did the work for him take?
- (11) Your revenue share of 1.1% was CHF 2'850. But actually 1.5 % was agreed.  
 a. How much must your employer pay you? b. Your goal for the next month is a commission of CHF 4'000. What turnover do you have to achieve?
- (12) In the first working year after your studies you will earn CHF 64,200. In the second year, your salary will rise to CHF 68,800.  
 a. What percentage is the salary increase after the first year.

- b. How much do you earn in the third year when your salary will increase by the same percentage?
- (13) You write off your car, which you bought for €16'500.-, annually by 12% of the residual value.
- How much is the value after one year?
  - What is the value after two years?
  - What is the value after 5 years?
- (14) You order replacement parts in Mexico, where they cost 2,500 pesos. The exchange rate is 0.1103 CHF/peso. How much CHF do these parts cost you?
- (15) On the highway from Châtel-St. Denis to Vevey there is a traffic sign indicating a 6% gradient. What does this mean in terms of distances?
- (16) In the sports store you buy a pair of gloves for CHF 100 including VAT. How much of this goes to the tax authorities if the VAT is 7.7%? (Attention, what does 7.7% VAT mean?)
- (17) In the department store you can find different soaps in different packages and prices. The first weighs 388g and costs CHF 4, the second 555g and costs CHF 5.35 and the third weighs 400g at CHF 4.20.
- How much does 100g of these three products cost?
  - How much soap do you get per CHF?
  - Indicate the relative price difference in percent.
- (18) You make a bike tour of 45 km. After 1h15 you have ridden 25 km. How long do you have left if you continue riding at the same speed?
- (19) We state the gasoline consumption of a car in  $\ell/100\text{ km}$ . In the USA people are interested in how many miles they can go with one gallon of gasoline. You know that one gallon is 3.78609  $\ell$  and one mile is 1.60934  $\text{km}$ .
- How many miles does your car go per gallon when it consumes 6  $\ell/100\text{ km}$ ?
  - Your friend's car drives 12  $\text{miles/gal}$ , how many liters does he need to drive 100  $\text{km}$ ?



Solutions

- (1) a. 7.2 b. 1.55 c. 0.55 d. 1.07
- (2) CHF 14.8
- (3) 27.27%
- (4) €34.49
- (5) 8
- (6) a. Math 4.75, French 5.14, English 5.14, Accounting 3.10  
b. Math 5.25, French 4.86, English 4.86, accounting not possible
- (7) CHF 1'563.25
- (8) 12.45 Births per thousand inhabitants
- (9) a. That leaves \$ 656.30, that is CHF 568.55  
b. One needed \$ 2650 / CHF 2431.45. On average one paid CHF 0.9175 / \$.
- (10) a. CHF 31.5  
b.  $2\frac{1}{4}$  h
- (11) a. CHF 1036.36  
b. CHF 266,666.6
- (12) a. 7.165%  
b. CHF 73'729.6
- (13) a. € 14,520.- b. € 12777.6 c. € 8,707.6
- (14) CHF 275.75
- (15) 6m/100m
- (16) CHF 7.15
- (17) a. CHF 1.03 CHF 0.96 CHF 1.05  
b. 97g 103.7g 95.24g  
c. 107% 100% 109%
- (18) 1 h
- (19) a. 39.21 miles/gal  
b. 19.6 l/100 km

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